Improving IV Estimates When the Direction of Endogeneity Bias is Known

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Outline

Motivation and Literature

Estimation in the Normal Experiment

Inference in the Normal Experiment

Asymptotic Results

Empirical Applications

Motivation

- In many settings, researchers may have strong prior beliefs about endogeneity bias.
- Examples:
  - Omitted variables: regression of income on education with unobserved ability $\implies$ OLS biased upwards;

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  - Selection: judges are more likely to send "bad guys" to prison than EM $\implies$ OLS biased downward;

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  - Simultaneity: preexisting skills facilitate new technology and new technology attracts skills $\implies$ OLS biased upwards;
  - Selection: judges are more likely to send "bad guys" to prison than EM $\implies$ OLS biased downward;
- Such beliefs are typically not incorporated in statistical analysis.

Motivation

- Let β and γ denote the IV and OLS estimands; $\hat{\beta}_n$ and $\hat{\gamma}_n$ denote the corresponding estimators.
- With strong IVs, for large n:

$$\begin{bmatrix} \hat{\beta}_n \\ \hat{\gamma}_n \end{bmatrix} \overset{d}{\approx} N\left(\begin{bmatrix} \beta \\ \gamma \end{bmatrix}, \begin{bmatrix} \hat{\sigma}^2_\beta & \hat{\sigma}_{\beta\gamma} \\ \hat{\sigma}_{\beta\gamma} & \hat{\sigma}^2_\gamma \end{bmatrix} \right).$$

  Additionally, know that $\beta \leqslant \gamma$.

- Typically $\hat{\sigma}^2_\beta \gg \hat{\sigma}^2_\gamma$, while $|\hat{\gamma}_n - \hat{\beta}_n|$ is often relatively small.
- As a result, $\hat{\gamma}_n$ should be informative about β.

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How to use this information in statistical analysis?

Related Literature: Econometrics

- Extra inequality restrictions:
  - Moon, Schorfheide (2009);
  - Andrews, Armstrong (2017);
  - Ketz, McCloskey (2022);
  - Cox (2024);
- Inference with nuisance parameters under the null:
  - Moreira (2003), Ketz (2018);
  - Romano, Shaikh, Wolf (2014);
  - Elliott, Müller, Watson (2015), Montiel Olea (2018);
- Parameters on the boundary:
  - Andrews (1999; 2001); McCloskey (2012);
  - Andrews, Guggenberger (2010); AG & Cheng (2011)
  - Fang and Santos (2019); Hong and Li (2020)

Related Literature: Statistics

- Estimation with restricted parameter spaces:
  - Lee (1981); Kelly (1989); Hwang, Peddada (1994); Peddada, Dunson, Tan (2005); Chang, Fukuda, Shinozaki (2017);
  - Marchard, Strawderman (2004); van Eden (2006);
- Inference with restricted parameter spaces:
  - Perlman, Wu (1999);
  - Silvapulle, Sen (2005);
  - Lehman, Romano (2005);

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Estimation in the Normal Experiment

- Let $X \sim N(\theta, \Sigma)$ in $\mathbb{R}^2$ with $\theta_1 \leqslant \theta_2$ and known Σ.
- The goal is to estimate $\theta_1 \in \mathbb{R}$.
- Unbiased MLE is X_1. In the constrained experiment, it is still minimum-variance unbiased and minimax.

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- The goal is to estimate $\theta_1 \in \mathbb{R}$.
- Unbiased MLE is X_1. In the constrained experiment, it is still minimum-variance unbiased and minimax.
- A natural alternative is Restricted MLE, defined as:

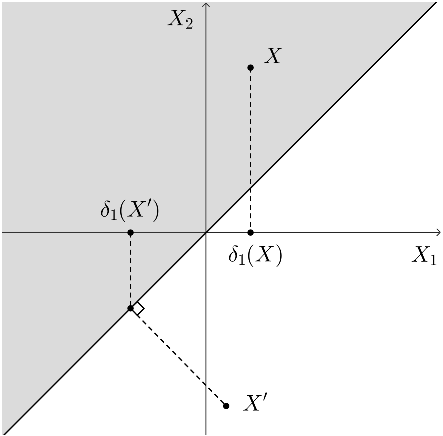
$$(\delta_1(X), \delta_2(X)) = \underset{\theta \in \mathbb{R}^2}{\operatorname{argmin}}\{(X-\theta)'\Sigma^{-1}(X-\theta) \mid \theta_1 \leqslant \theta_2\}.$$

- The solution is:

$$\delta_1(X) = X_1 \cdot \mathbf{1}(X_1 \leqslant X_2) + (w_1 X_1 + w_2 X_2) \cdot \mathbf{1}(X_1 > X_2),$$

where $w_2 = (\sigma_1^2 - \sigma_{12})/(\sigma_1^2 + \sigma_2^2 - 2\sigma_{12})$, and $w_1 = 1 - w_2$.

Illustration: RMLE



RMLE as an Estimator

Lemma 1

1. *If* $\sigma_1^2 \neq \sigma_{12}$*, then for all* $t > 0$ *and* $\theta_1 \leqslant \theta_2$*,*

$$P(|\delta_1(X) - \theta_1| \leqslant t) > P(|X_1 - \theta_1| \leqslant t).$$

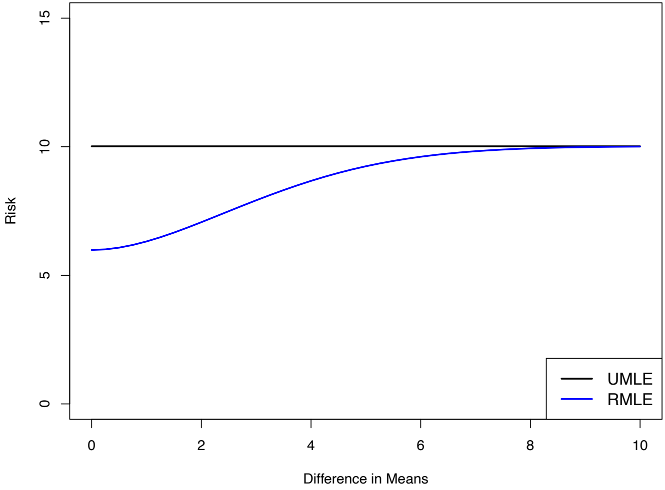
   Thus, $\delta_1(X)$ *dominates* X_1 *for any loss function* $L(x) = \rho(|x|)$*, where* $\rho : \mathbb{R}_+ \to \mathbb{R}_+$ *is non-decreasing.*

2. *If* $\sigma_1^2 > \sigma_{12}$*,* $\delta_1(X)$ *is biased downwards. For* $a < 0$*,* $b > 0$*,*

$$P(a \leqslant \delta_1(X) - \theta_1 \leqslant b) > P(a \leqslant X_1 - \theta_1 \leqslant b)$$

   holds for all $\theta_1 \leqslant \theta_2$ *if and only if* $a + b \geqslant 0$*. Thus,* $\delta_1(X)$ *dominates* X_1 *for* $L(x) = \rho_1(|x|)\mathbf{1}(x > 0) + \rho_2(|x|)\mathbf{1}(x \leqslant 0)$*, where* $\rho_j : \mathbb{R}_+ \to \mathbb{R}_+$ *are non-decreasing and* $\rho_1(t) \geqslant \rho_2(t)$*.*

Risk Profiles



Note: $\Sigma = \begin{bmatrix} 10 & 1 \\ 1 & 2 \end{bmatrix}$; quadratic loss.

Discussion

- Lemma 1 implies UMLE is inadmissible for many loss functions.
- Also implies that RMLE is minimax.
- van Eden and Zidek (2002) show RMLE itself is inadmissible under quadratic loss if Σ is diagonal, but:
  - The dominators are very complicated / unusable for inference;
  - Risk improvement over RMLE is marginal;
- Room for extensions in higher dimensions:
  - For diagonal Σ, completely solved by Kelly (1989);
  - For certain non-diagonal Σ, Hwang, Peddada (1994) show RMLE behaves unexpectedly in dimensions eight and higher;
  - For specific Σ, estimators dominating UMLE have been proposed;

Extensions

- Consider an experiment $X \sim N(\theta, \Sigma)$ in $\mathbb{R}^k$ with a target parameter:

$$\mu_1 = t'\theta$$

  and a single inequality constraint:

$$c'\theta \geqslant b.$$

- Idea: can reformulate the problem as as

$$Y \sim N(\mu, \sigma^2 I) \in \mathbb{R}^2$$

  with $\mu = (\mu_1, \mu_2)$ and $a'\mu \geqslant 0$.
- Can still show universal dominance of RMLE over MLE!

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RMLE for Inference

- Testing $H_0 : \theta_1 = t$ against $H_1 : \theta_1 \neq t$.
- The standard t-test with acceptance region

$$\{(x_1, x_2) \in \mathbb{R}^2 : |x_1 - t| \leqslant z_{1-\alpha/2}\sigma_1\}$$

is still UMP unbiased but ignores the implications of Lemma 1.

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  is still UMP unbiased but ignores the implications of Lemma 1.
- Instead, consider a test with acceptance region:

$$\{(x_1, x_2) \in \mathbb{R}^2 : |\delta_1(x) - t| \leqslant c(x)\}.$$

  By Lemma 1, $c(x) = z_{1-\alpha/2}\sigma_1$ yields a valid but conservative test.

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  By Lemma 1, $c(x) = z_{1-\alpha/2}\sigma_1$ yields a valid but conservative test.
- **Problem:** the distribution of $\delta_1(X) - t$ under H_0 is the same as:

$$g(Z; h) = Z_1 + w_2 \min(Z_2 - Z_1 + h, 0),$$

  where $Z \sim N(0, \Sigma)$ in $\mathbb{R}^2$, $h = \theta_2 - \theta_1$, and $w_2 = \frac{\sigma_1^2 - \sigma_{12}}{\sigma_1^2 + \sigma_2^2 - 2\sigma_{12}}$.

Nuisance Parameters Under the Null

- Well-known approaches:
  - Least-favorable tests;
  - Conditional tests: Moreira (2003); Ketz (2018);
  - Pretesting: Romano, Shaikh, Wolf (2014);
  - Max-WAP: Elliott, Müller, Watson (2015), Montiel Olea (2018);

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- **Proposed solution:**
  - A test with acceptance region $\{x : |\delta_1(x) - t| \leqslant c(x)\}$
  - Setting $c = Q_{1-\alpha}(|g(Z; h)|)$ would give a valid test;
  - The distribution of $|g(Z; h)|$ is stochastically increasing in h;
  - Choose data-dependent $h(X)$ to match the desired level;

The Test

- Set:

$$h_{t,\kappa}(X) = X_2 - X_1 + \tfrac{\sigma_1^2 - \sigma_{12}}{\sigma_1^2}(X_1 - t) + \kappa,$$

where $\kappa \geqslant 0$ is the smallest value such that

$$\sup_{h \geqslant 0} P\left\{|g(Z,h)| \geqslant Q_{1-\alpha}^{\tilde{Z}|Z}\left(|g(\tilde{Z}; s_\kappa(Z))|\right)\right\} \leqslant \alpha,$$

for $Z, \tilde{Z}$ independent $N(0, \Sigma)$ and

$$s_\kappa(Z) = \max(Z_2 - Z_1 + h + \tfrac{\sigma_1^2 - \sigma_{12}}{\sigma_1^2} Z_1 + \kappa, 0).$$

- Define:

$$\phi^{RMLE}(X;t) = \mathbf{1}\left\{\ |\delta_1(X) - t| > Q_{1-\alpha}^{Z|X}(|g(Z, h_{t,\kappa}(X) \vee 0)|)\ \right\}.$$

Two-Sided Tests with RMLE

Lemma 2 (Proposed Test)

Consider testing $H_0 : \theta_1 = t$ against $H_1 : \theta_1 \neq t$. Let $h_{t,\kappa}(X)$ and κ be as defined above. Then, the test $\phi^{RMLE}(X; t)$ is level-α.

Two-Sided Tests with RMLE

Lemma 2 (Proposed Test)

Consider testing $H_0 : \theta_1 = t$ *against* $H_1 : \theta_1 \neq t$. *Let* $h_{t,\kappa}(X)$ *and* κ *be as defined above. Then, the test* $\phi^{RMLE}(X;t)$ *is level-*α.

Lemma 3 (Conditional LR Test)

Consider testing $H_0 : \theta_1 = t$ *against* $H_1 : \theta_1 \neq t$. *The test:*

$$\phi^{CLR}(X) = \mathbf{1}\{|\delta_1(X) - t| > Q_{1-\alpha}^{Z|X}(|g(Z; h_{t,0}(X))|)\},$$

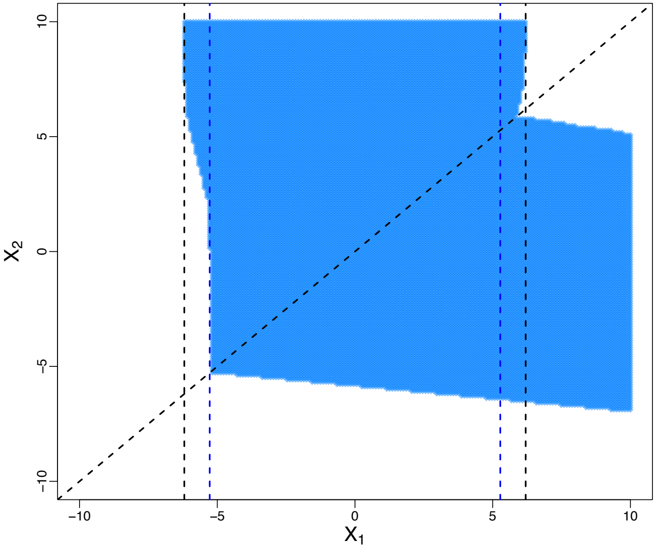
where $Z = (Z_1, \frac{\sigma_{12}-\sigma_1^2}{\sigma_1^2} Z_1)$ *with* $Z_1 \sim N(0, \sigma_1^2)$, *is level-*$\alpha$.

Discussion

- Both tests are biased w.r.t. alternatives $\theta_1 > t$.
- Both CIs centered around $\delta_1(X)$; not nested with $[X_1 \pm z_{1-\alpha/2}\sigma_1]$.
- RMLE CI is shorter than $2z_{1-\alpha/2}\sigma_1$ with probability one.
- CLR CI may not be.
- Parameter κ for RMLE is easy to compute numerically.

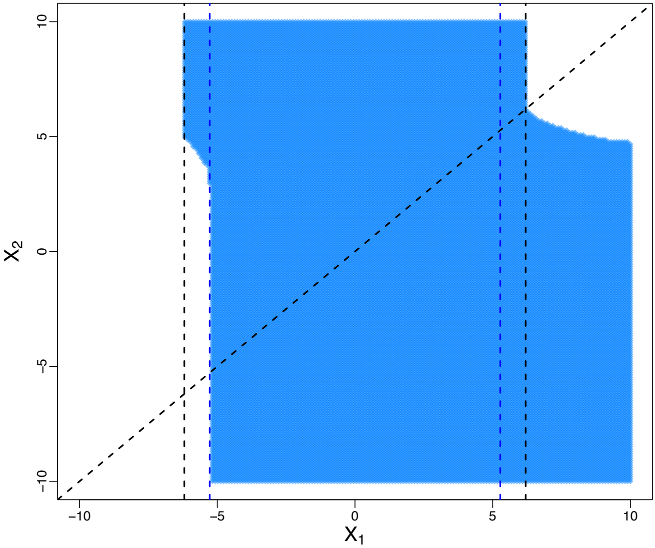
RMLE Acceptance Region

Testing $H_0 : \theta_1 = 0$ vs $H_1 : \theta_1 \neq 0$

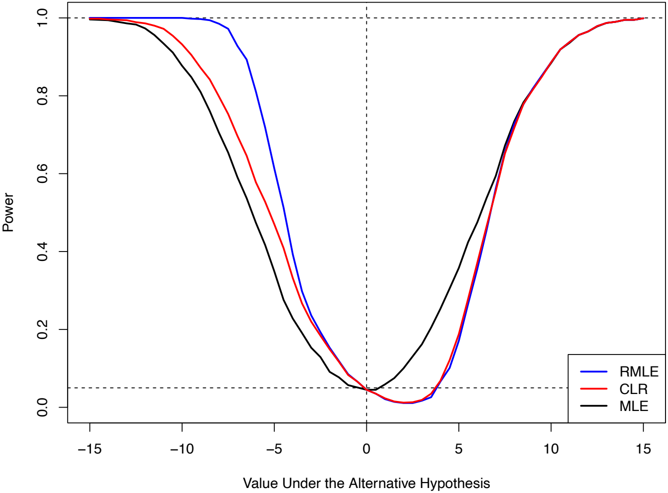


CLR Acceptance Region

Testing $H_0 : \theta_1 = 0$ vs $H_1 : \theta_1 \neq 0$

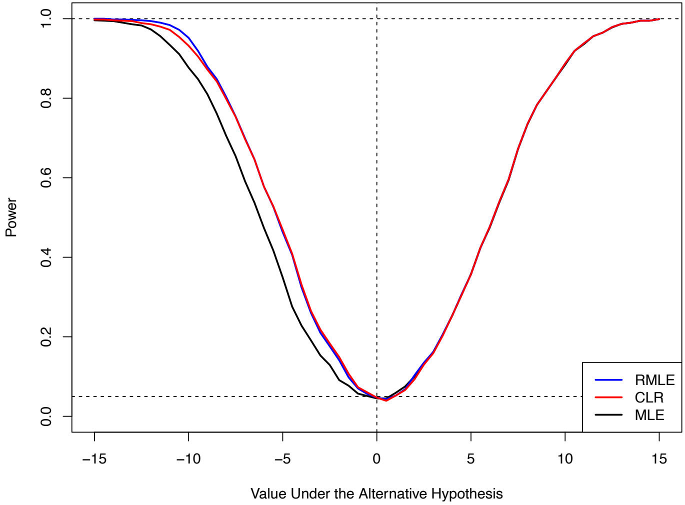


Power Comparisons



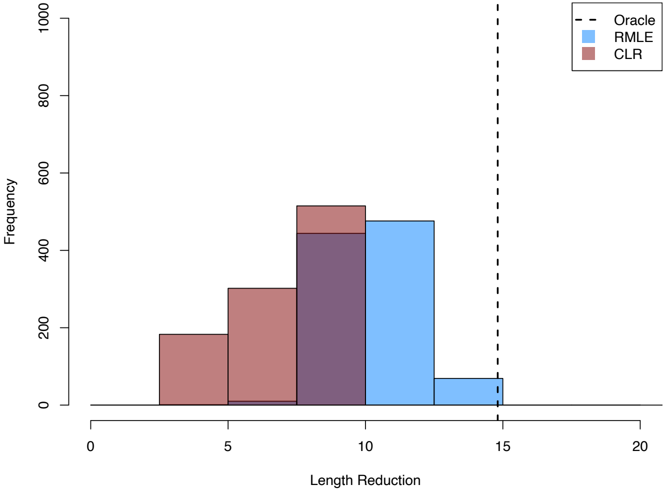
Note: $\Sigma = \begin{bmatrix} 10 & 1 \\ 1 & 2 \end{bmatrix}; \quad h = 0.$

Power Comparisons



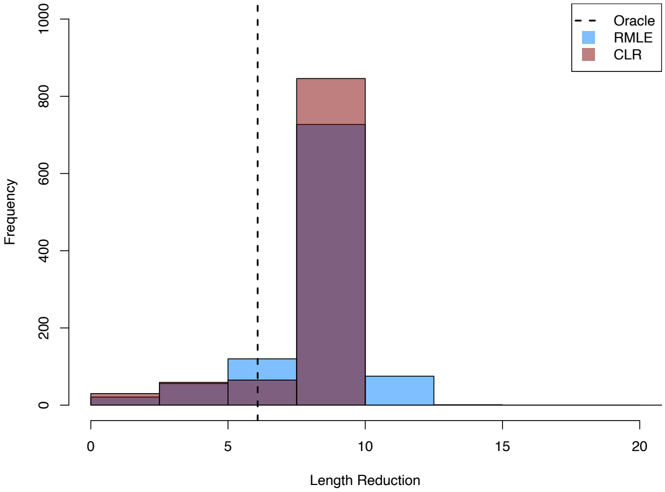
Note: $\Sigma = \begin{bmatrix} 10 & 1 \\ 1 & 2 \end{bmatrix}; \quad h = 5.$

% Reduction of CI Length



Note: Σ as above, $h = 0$. Median for RMLE is 11%, max is 14%.

% Reduction of CI Length



Note: Σ as above, h $=$ 5. Median for RMLE is 8%, max is 13%.

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Setup

Assumption 1

The distribution P of the data belongs to the set:

$$\mathbf{P} = \{P \in \mathcal{M} : \beta(P) \leqslant \gamma(P)\},$$

where $\mathcal{M}$ is defined by suitable moment and support restrictions.

Assumption 2

There is a sequence of regular estimators $(\hat{\beta}_n, \hat{\gamma}_n)$ satisfying

$$\begin{bmatrix} \sqrt{n}(\hat{\beta}_n - \beta) \\ \sqrt{n}(\hat{\gamma}_n - \gamma) \end{bmatrix} \overset{d}{\to} N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_\beta^2 & \sigma_{\beta\gamma} \\ \sigma_{\beta\gamma} & \sigma_\gamma^2 \end{bmatrix} \right), \tag{1}$$

with non-degenerate covariance matrix Σ, and a sequence of estimators $\hat{\Sigma}_n$ satisfying $\hat{\Sigma}_n \to_p \Sigma$, for each $P \in \mathbf{P}$.

Setup

- The estimator:

$$
\begin{aligned}
\hat{\delta}_n &= \hat{\beta}_n \mathbf{1}(\hat{\beta}_n \leqslant \hat{\gamma}_n) + (\hat{w}_{1,n}\hat{\beta}_n + \hat{w}_{2,n}\hat{\gamma}_n)\mathbf{1}(\hat{\beta}_n > \hat{\gamma}_n) \\
&= \hat{\beta}_n + \hat{w}_{2,n} \min(\hat{\gamma}_n - \hat{\beta}_n, 0),
\end{aligned}
$$

  where $\hat{w}_{2,n} = (\hat{\sigma}^2_{\beta,n} - \hat{\sigma}_{\beta\gamma,n})/(\hat{\sigma}^2_{\beta,n} + \hat{\sigma}^2_{\gamma,n} - 2\hat{\sigma}_{\beta\gamma,n})$.

- Local asymptotics: let $P_{n,h} \in \mathbf{P}$ be a sequence of distributions local to P such that $\sqrt{n}(\gamma(P_{n,h}) - \beta(P_{n,h})) \to h$ for some $h \geqslant 0$.

Asymptotic Properties: Estimation

Theorem 1

Let Assumptions 1 and 2 hold and $P_{n,h}$ as above. Then:

1. *Along $P_{n,h}$:*

$$\sqrt{n}(\hat{\delta}_n - \beta_{n,h}) \quad \to_d \quad Z_1 + w_2 \min(Z_1 + Z_2 + h, 0),$$

   where $Z = (Z_1, Z_2) \sim N(0, \Sigma)$.

2. *For any $h > 0, t > 0$, there exists $N(t,h) \in \mathbb{N}$ such that:*

$$P_{n,h}(|\sqrt{n}(\hat{\delta}_n - \beta_{n,h})| \leqslant t) > P_{n,h}(|\sqrt{n}(\hat{\beta}_n - \beta_n)| \leqslant t)$$

   for all $n \geqslant N(t,h)$.

Asymptotic Properties: Inference

Theorem 2

Let Assumptions 1 and 2 hold, $H_0 : \beta(P) = \beta_0$ and $H_1 : \beta(P) \neq \beta_0$. Define:

$$\bar{h}_n = \sqrt{n}(\hat{\gamma}_n - \hat{\beta}_n) + \frac{\hat{\sigma}^2_{\beta,n} - \hat{\sigma}_{\beta\gamma,n}}{\hat{\sigma}^2_{\beta,n}}\sqrt{n}(\hat{\beta}_n - \beta_0) + \hat{\kappa}_n,$$

where $\hat{\kappa}_n \geqslant 0$ is the smallest value such that:

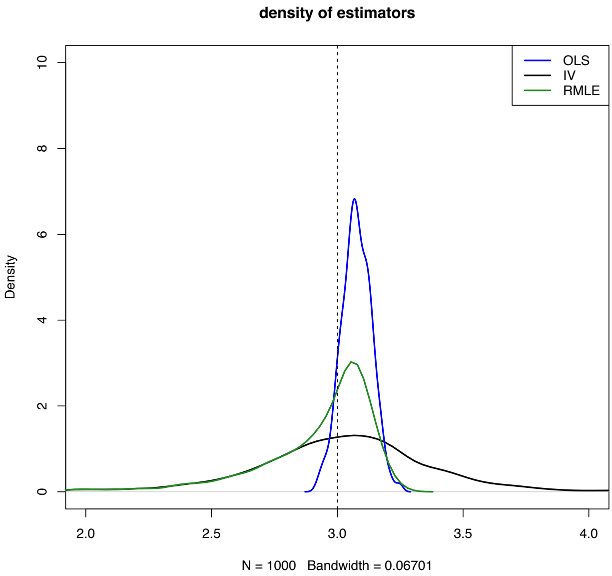
$$\sup_{h \geqslant 0} P\left(|g(Z,h)| \geqslant Q^{\tilde{Z}|Z}_{1-\alpha}(|g(\tilde{Z}; s_\kappa(Z))|)\right) \leqslant \alpha,$$

with $Z, \tilde{Z}$ i.i.d. $N(0, \hat{\Sigma}_n)$, conditional on the data. The test:

$$\hat{\phi}_n(t) = \mathbf{1}(\sqrt{n}|\hat{\delta}_n - \beta_0| \geqslant Q^{Z|W_1^n}_{1-\alpha}(|g(Z; \bar{h}_n \vee 0)|)),$$

is locally asymptotically level-α.

A Snapshot from Simulations



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Criminal Recidivism and Electronic Monitoring

Di Tella and Schargorodsky (2013)

- Argue that using EM instead of prison leads to lower recidivism.
- Using individual-level data, estimate:

$$R_i = \alpha + \beta D_i + X_i'\eta + \varepsilon_i$$

  - R_i — indicator for recidivism;
  - D_i — indicator for being assigned an EM;
  - X_i — {type of crime, age, nationality, time dummies, ...}

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  - X_i — {type of crime, age, nationality, time dummies, ...}

- **Endogeneity bias:** the "bad guys" are less likely to get out with an EM $\implies$ OLS is biased upwards.
- **Instrument:** judge characteristics.

Criminal Recidivism and Electronic Monitoring

Results

- Re-running the main specification yields:

$$\begin{bmatrix} \hat{\beta}_n^{IV} \\ \hat{\gamma}_n^{OLS} \end{bmatrix} = \begin{bmatrix} -0.137 \\ -0.090 \end{bmatrix}; \;\; \hat{\Sigma}_n = \begin{bmatrix} 4.965 & 0.597 \\ 0.597 & 0.410 \end{bmatrix},$$

  with $n = 1503$. Expect $\beta \leqslant \gamma$ and get $\hat{\beta}_n^{IV} \leqslant \hat{\gamma}_n^{OLS}$.
- The implied 95% IV confidence interval is $[-0.245, -0.020]$.

Criminal Recidivism and Electronic Monitoring

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  with $n = 1503$. Expect $\beta \leqslant \gamma$ and get $\hat{\beta}_n^{IV} \leqslant \hat{\gamma}_n^{OLS}$.
- The implied 95% IV confidence interval is $[-0.245, -0.020]$.
- $\hat{\delta}_n^{RMLE} = -0.137$ produces $[-0.245, -0.040]$, which is 9% shorter.
- Despite the large sample size, good IVs, and the direction of the bias as expected, still see an improvement.

Mechanics of Industrial Revolution

Kelly, Mokyr, Ó Gráda (2023)

- Argue that different patterns of growth across counties of England in 1760s–1830s can be explained by pre-existing supply of mechanical skills.
- Using county-level data, estimate:

$$Y_i = \alpha + D_i\beta + X_i'\eta + \varepsilon_i$$

  - Y_i — share of men working in textiles in 1831 (or 1851);
  - D_i — existing supply of mechanical skills based on 1790s;
  - X_i — average wages in 1760s and market potential in 1750s;

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  - D_i — existing supply of mechanical skills based on 1790s;
  - X_i — average wages in 1760s and market potential in 1750s;
- **Endogeneity bias:** new textile industries may have attracted skilled workers, not vice versa $\Longrightarrow$ OLS is biased upwards.
- **Instrument:** apprentice fees.

Mechanics of Industrial Revolution

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- Re-running the main specification yields:

$$\begin{bmatrix} \hat{\beta}_n^{IV} \\ \hat{\gamma}_n^{OLS} \end{bmatrix} = \begin{bmatrix} 3.178 \\ 2.125 \end{bmatrix}; \;\; \hat{\Sigma}_n = \begin{bmatrix} 72.762 & 8.448 \\ 8.448 & 7.577 \end{bmatrix},$$

  with $n = 41$. Expect $\beta \leqslant \gamma$ but get $\hat{\beta}_n^{IV} > \hat{\gamma}_n^{OLS}$.

- The implied 95% IV confidence interval is $[0.566, 5.789]$.

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  with $n = 41$. Expect $\beta \leqslant \gamma$ but get $\hat{\beta}_n^{IV} > \hat{\gamma}_n^{OLS}$.

- The implied 95% IV confidence interval is $[0.566, 5.789]$.
- $\hat{\delta}_n^{RMLE} = 2.11$ yields $[-0.309, 4.299]$, which is 12% shorter and includes zero.
- Similar results for 1851 specification.

Learning by Doing

Levitt, List, Syverson (2013)

- Document learning by doing at an automobile assembly plant.
- Using daily and weekly data, estimate:

$$\log S_t = \alpha + \beta \log E_t + \varepsilon_t$$

  - S_t — quality of produced cars;
  - E_t — production experience at time t.

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  - S_t — quality of produced cars;
  - E_t — production experience at time t.

- **Endogeneity bias:** manager steers production towards the higher-quality shift $\implies$ OLS is biased downwards.
- **Instrument:** experience of the other shift.

Learning by Doing

Results

- Re-running the main specification using weekly data yields:

$$\begin{bmatrix} \hat{\beta}_n^{IV} \\ \hat{\gamma}_n^{OLS} \end{bmatrix} = \begin{bmatrix} -0.282 \\ -0.267 \end{bmatrix}; \;\; \hat{\Sigma}_n = \begin{bmatrix} 0.065 & 0.055 \\ 0.055 & 0.047 \end{bmatrix},$$

  with $n = 39$. Expect $\beta \geqslant \gamma$ but get $\hat{\beta}_n^{IV} < \hat{\gamma}_n^{OLS}$.
- The implied 95% IV confidence interval is $[-0.363, -0.203]$.

Learning by Doing

Results

- Re-running the main specification using weekly data yields:

$$\begin{bmatrix} \hat{\beta}_n^{IV} \\ \hat{\gamma}_n^{OLS} \end{bmatrix} = \begin{bmatrix} -0.282 \\ -0.267 \end{bmatrix}; \quad \hat{\Sigma}_n = \begin{bmatrix} 0.065 & 0.055 \\ 0.055 & 0.047 \end{bmatrix},$$

  with $n = 39$. Expect $\beta \geqslant \gamma$ but get $\hat{\beta}_n^{IV} < \hat{\gamma}_n^{OLS}$.

- The implied 95% IV confidence interval is $[-0.363, -0.203]$.
- $\hat{\delta}_n^{RMLE} = -0.227$ produces $[-0.295, -0.156]$, which is 13% shorter, and the difference is economically significant:
  - $\hat{\beta}_n^{IV}$ implies the rate of defects is halved for every 12-fold increase in production;
  - $\hat{\delta}_n^{RMLE}$ implies that for 21-fold;
- Similar results using daily data with $n = 190$ observations.

Summary

- Knowing the direction of endogeneity bias can meaningfully improve statistical analysis.
- This paper proposed a simple way to construct more precise estimates and shorter confidence intervals.
- Obtained 9-13% shorter CIs in three empirical applications.
- Further research: minimum expected length CI? extensions to multiple inequalities? applications to partial identification?